LAB # 9: MOMENTS OF INERTIA

Introduction: The inertia of a given body is defined by mass. Inertia, or the resistance of an object to a change in its linear motion, is directly proportional to the mass of that object. In other words, the greater the mass, the greater the inertia. In a similar way, the resistance to change in angular motion is determined by the **moment of inertia (I)**, which includes the mass of the object. However it also depends upon the distribution of the mass with respect to the axis of rotation. If the mass of an object is concentrated close to the axis of rotation, then the moment of inertia is smaller and that object is easier to rotate than if the mass is concentrated farther away from the axis. This is important when objects can change the arrangement of mass (such as a human body changing shape) or when an object may rotate around more than one axis. A simple illustration of this point is the swinging of a baseball bat. When a bat is swung from the proper "grip" end, the resistance to change in angular motion is much greater than when it is swung from the opposite "barrel" end. Why? Because the mass of the bat is concentrated closer to the axis of rotation (the grip or wrists) when it is swung from the "barrel" end. Thus, the resistance to change in angular motion is considerably less, and the batter can more easily start and stop the bat's rotation.

If the shape of the object is irregular, the distribution of mass with respect to the axis of rotation is difficult to calculate theoretically, so various experimental methods are used to estimate the moment of inertia. One such method involves a <u>pendulum swing technique</u> and is the focus of today's laboratory exercise. The formula for determining ***I*** in this instance is as follows.

$$I_{xx} = \left(\frac{t}{2\pi}\right)^2 \times m \times g \times L$$

where:	I_{xx}	=	moment of inertia about axis of rotation [$kg \cdot m^2$]
	t	=	time period required for one COMPLETE swing [s]
	m	=	the mass of the object [kg]
	g	=	acceleration due to gravity [m/s^2]
	L	=	the distance from axis of rotation to the center of mass [m]

Equipment: Stopwatch, meter stick, scale, clamps, objects to swing

Procedure: This lab is divided into three swing testing stations and two measurement sites. Three different pieces of sporting equipment will be tested, one around two different axes. The class will be divided into FOUR groups, and each group will determine the moment of inertia values of the three implements, using the above formula and the pendulum swing technique. Each group will need to use a stopwatch, a metric ruler and a scale for the data collection.

To collect data, suspend an object from the clamps using the metal rod through one end of the object. Then swing the object gently and time the oscillation period. Repeat for rod (axis) in each implement.

Each group needs collect data on four "axes" and fill out the table on the next page. Rotate the measurement responsibilities among your group members so that everyone participates in the data collection.

Time [s]: When timing the swing of each object, ***the swing angle must not exceed 5 ^{o} each way***. Time a series of at least 7 to 10 COMPLETE pendulum oscillations. Each oscillation (swing) includes a forward and a return phase. Then divide the total time by the number of oscillations to get the average time per oscillation (swing).

Length [m]: The distance from the axis of rotation to the point where the object's center of mass is located (balance point). You need to determine the object's balance point along its long axis. You will need to decide how to place the object so that you can balance it reliably.

Gravity: Acceleration due to gravity = ___________ m/s^2

Mass: Make sure you use kg units for mass during your calculation. Convert units as necessary.

<u>Data Table:</u>

Swings = total number of swings
Total Time = total time of swinging
Avg. Time = (total swing time) / (# of swings)
m [kg] = mass of object
g [m/s^2] = acceleration due to gravity (9.81 m/s^2)
L [m] = distance from axis of rotation to the object's COM
I_{xx} [$kg \cdot m^2$] = moment of inertia about the axis tested – to be calculated from all the above variables.

$$I_{xx} = \left(\frac{t}{2\pi}\right)^2 \times m \times g \times L$$

OBJECT	**Axis**	**# Swings**	**Total Time**	**Avg. Time**	**m [kg]**	**g [m/s^2]**	**L [m]**	**I_{xx} [$kg \cdot m^2$]**

Show your calculations for obtaining I_{xx} for one of the objects:

I_{xx} gives you the moment of inertia relative to the axis about which the object is swinging. But often we need to know the moment of inertia about an axis through the object's center of mass. Complete the table below using the following formula to calculate the moment of inertia about a transverse axis through the center of mass (I_{cm}) for each object.

$$I_{xx} = I_{cm} + mL^2$$

where:

	I_{xx}	=	moment of inertia about non-centered axis of rotation
	I_{cm}	=	moment of inertia about axis through the center of mass
	m	=	the mass of the object
	L	=	the distance from axis of rotation to the center of mass

Copy data from the previous chart into the chart below so that you can calculate I_{cm}

OBJECT	Axis	I_{xx} [kg·m^2]	m [kg]	L [m]	I_{cm} [kg·m^2]

Show your calculations for obtaining I_{cm} for one of the objects:

Question: Compare your values for I_{xx} for each object from its grip. How and why do they differ?

Question: Compare your values for I_{xx} for the bat from its two ends. How and why do they differ?

Question: Compare the I_{cm} of the bat calculated from the two ends.

1. Do the two values match?
2. Should they match?
3. Why or why not?

Question: State at least three experimental assumptions that were made to estimate the moment of inertia I.

Question: Coaches often advise athletes to change the location of their grip on a striking implement to improve performance. How do the data from this lab suggest moving the hand grip location along the handle toward the striking point would affect performance?